In the case of a constant stress σ_1 applied to the boundary of a rectangular area

$$\psi_1(\zeta_R, \xi_R) = k_R^{-2} \sin^{-1}\varphi \cos^{-1}\varphi \sin(k_R a \cos\varphi) \sin(k_R b \sin\varphi) \sigma_1.$$

Hence it follows that if the size of the area is selected according to the dependences $a = n\pi v_R/\omega \cos\varphi$ or $b = n\pi v_R/\omega \sin\varphi$, then there will be no displacements. Therefore, the influence of the inhomogeneity of the medium on the size of the area for which the maximal or minimal part of the energy of the vibrations source is transmitted to excitation of a Rayleigh wave in a given direction can be estimated.

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ANTIPLANAR DEFORMATION OF AN ELASTOPLASTIC STRIP WITH A SEMI-INFINITE CRACK

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UDC 539.374

The features and details of plastic flow at a crack tip govern its development [1]. Therefore, it is important to have a correct idea about the shape and dimensions of the plastic zone, and about the intensity of deformation in it. In view of this there is considerable importance in the problem during whose solution, apart from determining stresses and strains, there should be determination without prior assumptions of the boundary separating the elastic and plastic regions. A study was made in [2-8] of approximate and numerical methods for this problem, and analytical solutions in closed form have only been obtained for antiplanar deformation of a boundless material with one rectilinear crack or a periodic system of collinear defects [9-14]. In this work an accurate solution is obtained for the elastoplastic problem of antiplanar deformation of a strip with a semi-infinite crack.

We consider antiplanar deformation of a strip made of elastoplastic material occupying the region $|x| < \infty$, $|y| \leq d$. It is assumed that in the plastic condition material behavior is described by the Tresk condition

$$\sigma_{xz}^2 + \sigma_{yz}^2 = \tau_*^2 \tag{1}$$

and by an associated rule for plastic flow, and in the elastic region by Hooke's linear rule

$$\sigma_{xz} = \mu \frac{\partial w}{\partial x}, \quad \sigma_{yz} = \mu \frac{\partial w}{\partial y}, \tag{2}$$

$$\sigma_{xz}^2 + \sigma_{yz}^2 < \tau_*^2, \tag{3}$$

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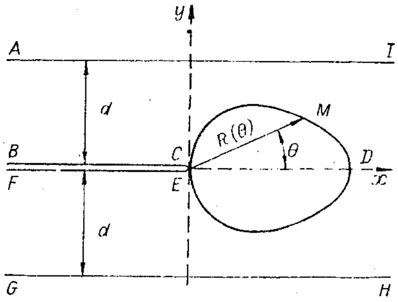

Fig. 1

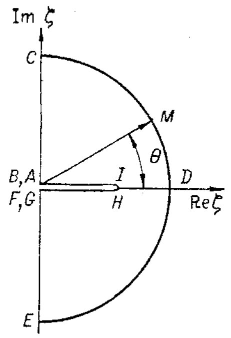

Fig. 2

where σ_{xz} and σ_{yz} are stress tensor components; τ_* is yield strength; μ is shear modulus; w is displacement along axis z. In a strip with y = 0, x < 0 there is a semiinfinite cut-crack whose sides are free from load

$$\sigma_{yz} = 0,\ x < 0,\ y = 0. \tag{4}$$

It is well known [5, 9] that with antiplanar deformation a region of plasticity adjoins the crack tip and is located as a whole in the region x > 0 as shown in Fig. 1, and stress and strain distribution in the plastic zone have the form

$$\sigma_{\theta z} = \tau_*,\quad \sigma_{rz} = 0,\quad \frac{\partial w}{\partial \theta} = \frac{\tau_*}{\mu} R(\theta),\quad \frac{\partial w}{\partial r} = 0. \tag{5}$$

Here $r^2 = x^2 + y^2$; $R(\theta)$ is distance from the origin to the elastoplastic boundary. The value of $R(\theta)$ is previously unknown, and should be determined in the course of solving the problem.

Let at the boundary of the strip with $y = \pm d$ displacement be prescribed

$$w(x, \pm d) = \pm d\tau_0/\mu,\ |x| < \infty, \tag{6}$$

where τ_0 is a constant with a stress dimension ($\tau_0 \le \tau_*$). From (2) and (6) it follows that

$$\sigma_{xz} = 0,\ |x| < \infty,\ y = \pm d; \tag{7}$$

$$\sigma_{yz} - i\sigma_{xz} \to \tau_0,\ x \to +\infty,\ |y| \leqslant d. \tag{8}$$

In constructing a solution in the elastic region relationships (4) and (7) serve as boundary conditions at the crack edges and at the boundary of the strip, and relationship (1) plays the role of a boundary condition at the elastoplastic boundary. After constructing a solution in the elastic region and determining the shape of the elastoplastic boundary, stress and strain distribution in the plastic region are found from Eq. (5).

It is well known [15] that in the elastic region $\sigma_{yz} - i\sigma_{xz}$ is an analytical function of x − iy and conversely

$$\frac{x - iy}{d} = f'(\zeta),\quad \zeta = \frac{\sigma_{yz} - i\sigma_{xz}}{\tau_*}. \tag{9}$$

Here $f(\zeta)$ is a function, analytical with all ζ, corresponding to stresses in the elastic region, and a prime indicates differentiation for the argument. From (3), (4), (7)-(9), it follows that the elastic region of plane (x, y) reflects a half-circle of unit radius with a cut along the material axis in plane ζ (Fig. 2). Points A, B, ..., H, I, M in Fig. 2 correspond to points A, B, ..., H, I, M in Fig. 1, and in Fig. 2 point A merges with B, F with G, and point H with I. From (5) it follows [10] that angle θ in Fig. 1 equals angle θ in Fig. 2 for any point M at the elastoplastic boundary.

We designate in terms of $\partial/\partial p$ differentiation along the arc length of contour A, B, ..., H, I in Fig. 2 with counterclockwise passage. From (9) we obtain

$$\frac{\partial f}{\partial p} = \frac{x - iy}{d}\frac{\partial \zeta}{\partial p}. \tag{10}$$

Using Figs. 1 and 2 and expression (10) it is possible to show [10] that $\partial \operatorname{Re} f/\partial p = 0$ in the section of the boundary BCDEF. Therefore

$$\operatorname{Re} f = 0,\ \zeta \in BCDEF, \tag{11}$$

where the integration constant is assumed to equal zero. Since y = ±d and $\partial \zeta / \partial p = \pm 1$ with $\zeta \in$ AIHG, then $\partial\ \mathrm{Im}\ f/\partial p = -1$ at both edges of the cut. Consequently

$$\mathrm{Im}\, f = \pm T \mp \zeta,\ \zeta \in AIHG. \tag{12}$$

Here $T = \tau_0/\tau_*$, and the upper and lower signs correspond to the upper and lower edge of the cut in Fig. 2.

Thus, in order to determine function $f(\zeta)$, which is analytical in the region within the curve A, B, ..., H, I, we have a mixed boundary problem (11) and (12). By direct verification it is possible to be certain that its solution takes the form

$$f(\zeta) = \frac{1}{\pi}\left[\frac{\ln(1-T^2\zeta^2)}{\zeta} - \zeta \ln\left(1-\frac{T^2}{\zeta^2}\right)\right] + \frac{T}{\pi}\ln\left[\frac{(\zeta-T)(1+T\zeta)}{(\zeta+T)(1-T\zeta)}\right],$$
$$f'(\zeta) = -\frac{1}{\pi}\left[\ln\left(1-\frac{T^2}{\zeta^2}\right) + \frac{\ln(1-T^2\zeta^2)}{\zeta^2}\right]. \tag{13}$$

For the position of the elastoplastic boundary from (9) we find [10] that

$$R(\theta)/d = e^{i\theta} f'(e^{i\theta}). \tag{14}$$

The amount of displacement at the elastoplastic boundary is obtained from (5) with integration for θ

$$W = \mu w\,[R(\theta), \theta]/d\tau_* = -i\,[f(e^{i\theta}) - f(1)]. \tag{15}$$

Equations (5), (9) and (13)-(15) give a solution of the elastoplastic problem for a strip with a semiinfinite crack.

By substituting (13) in (14) and (15) we have

$$\frac{R(\theta)}{d} = -\frac{2}{\pi}(\ln q \cos\theta - \varphi \sin\theta),$$
$$W = \frac{2}{\pi}(T\varphi + 2T\psi - \varphi\cos\theta - \ln q \sin\theta),$$
$$q = (1 - 2T^2\cos 2\theta + T^4)^{1/2},$$
$$\sin\varphi = \frac{T^2}{q}\sin 2\theta, \quad |\varphi| \leqslant \frac{\pi}{2},$$
$$\sin\psi = \frac{T\sin\theta}{(1+2T\cos\theta+T^2)^{1/2}}, \quad |\psi| \leqslant \frac{\pi}{2}. \tag{16}$$

Analysis of expression (16) indicates that the maximum value $R = R_0$ is observed with $\theta = 0$, and $W = W_0$ is observed with $\theta = \pi/2$, and

$$\frac{R_0}{d} = -\frac{2}{\pi}\ln(1-T^2),$$
$$W_0 = \frac{2}{\pi}\left[2T \arcsin\frac{T}{\sqrt{1+T^2}} - \ln(1+T^2)\right]. \tag{17}$$

Dependences for R_0/d and W_0 on dimensionless load T are given in Fig. 3 (curves 1 and 2, respectively). It follows from (17) that with small T, R_0 and W_0 are proportional to T^2. With $T \to 1$ in contrast to the case of a single crack of finite length in a limitless material [10], the value of R_0 reverts to infinity by a logarithmic rule, and not exponentially. In view of this we consider the case of T = 1 separately. Relationship (13) is simplified:

$$f(\zeta) = i(1-\zeta) + \frac{1}{\pi\zeta}[(1-\zeta^2)\ln(1-\zeta^2) + 2\zeta^2\ln\zeta],$$
$$f'(\zeta) = -i - \frac{1}{\pi}\left[\left(1+\frac{1}{\zeta^2}\right)\ln(1-\zeta^2) - 2\ln\zeta\right],$$

and expressions for $R(\theta)$ and W are reduced to the form

$$\frac{R(\theta)}{d} = \frac{2}{\pi}\left[\left(\frac{\pi}{2}-\theta\right)\sin\theta - \cos\theta\ln(2\sin\theta)\right],$$
$$W = 1 - \cos\theta + \frac{2}{\pi}[\theta\cos\theta - \sin\theta\ln(2\sin\theta)],$$
$$R_0 = \infty, \quad W_0 = 1 - \frac{2}{\pi}\ln 2, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}. \tag{18}$$

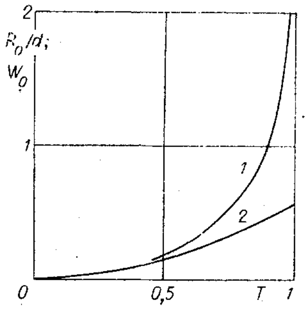

Fig. 3

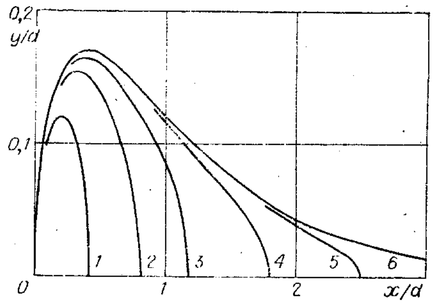

Fig. 4

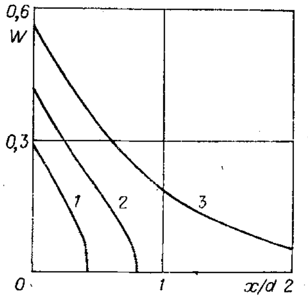

Fig. 5

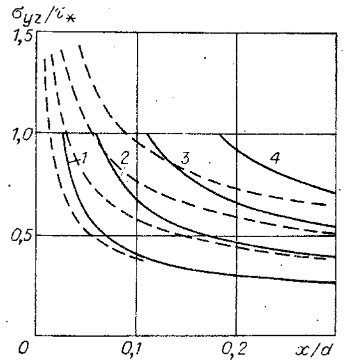

Fig. 6

The shape of the elastoplastic boundary calculated by Eqs. (16) and (18) is shown in Fig. 4 (curves 1-6 correspond to $T = 0.7$; 0.85; 0.92; 0.97; 0.99 and 1). With $T \to 0$ the plastic zone takes asymptotically the shape of a circle, which agrees with the result for the case of a single semiinfinite crack in a limitless material [5]. With $T \to 1$ plastic zone length is much greater than its width, i.e., with an increase in load the plastic region increases mainly in the direction of crack extension.

The amount of displacement at the elastoplastic boundary is given in Fig. 5, where curves 1-3 relate to $T = 0.7$, 0.85, and 1. In contrast to the case of a crack with finite length in a limitless material [10], displacement here at the elastoplastic boundary remains finite with $T \to 1$.

The dependence of stresses σ_{yz} on coordinate x with $y = 0$ (at the crack extension) is shown in Fig. 6 by solid lines 1-4 for $T = 0.2$; 0.3; 0.4 and 0.5, and broken lines with the same T give values of the function

$$\frac{\sigma_{yz}}{\tau_*} = \frac{T}{\sqrt{1 - e^{-\pi x/d}}}, \tag{19}$$

describing stress σ_{yz} distribution in the crack extension in the case of the elastic problem when the plastic zone at the crack tip is absent [16]. From the results presented in Fig. 6 it can be seen that relationships (9) and (19) only agree with $x > \Delta$, where $\Delta \sim R_0$. However, since $R_0 \sim T^2$ with $T \ll 1$, then in the case of a small load solution of the elastic problem with good accuracy approximates the solution of the elastoplastic problem of almost the whole strip $|y| \leq d$.

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CREEP OF AN ICE COATING LYING UPON A HYDRAULIC FOUNDATION UNDER THE ACTION OF A CONCENTRATED FORCE

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UDC 624.131+539.215

We consider an infinitely thin plate under cylindrical quasistatic inflection conditions lying upon a hydraulic foundation, i.e., a layer of viscous liquid of finite depth. The plate material is incompressible and such that the intensity of the deformation rate deviator and the stress deviator are related by a power dependence. Such relationships are often used to describe the stress-deformed state of constructions of ice under conditions of developed steady state creep. In the case of plate inflection by a concentrated force asymptotic expressions are found for deflection at short and long times. The solutions constructed make it possible to find the dependence of the deflection of an ice coating upon time as found in full-scale studies.

1. Formulation of the Problem. We will consider the case of cylindrical inflection of a plate of thickness h, lying upon a layer of incompressible viscous liquid of finite depth H (Fig. 1). The inflection is accomplished by a concentrated force Q. To perform experiments on ice covers a special ice-cutting apparatus was used, weighing 2.5 tons, about 7 m long, with a bearing surface of 0.13 m^2, and additional load weights from 2 to 30 tons. The thickness of the ice plate h = 0.25-0.35 m, with the thickness of the liquid layer H = 10-16 m.

We assume that the plate material can be described by Glen's law, which assumes a power relationship between the intensity of tangent stresses and the deformation rate [1], $\dot{\varepsilon} = B\sigma^b$, where B and b are a coefficient and the creep index (greater than unity). In solving the problem we will use the steady state creep equation, i.e., the plate's deflection should increase linearly with time. In reality, because of the interaction between the plate and the base its deflection proves to depend nonlinearly on time, but the linear term is the main one at both short and long relative time intervals.

The deflection of the plate can always be expressed in the form $y = y_e + y_c$, where the term y_e considers elastic and plastic deformations which develop practically instantaneously upon load application, while y_c de-

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